Dynamic Visualisations

A GeoGebra Workbook for Selected Topics in the Mathematics CAPS Curriculum, Grades 10-12.

Dr Philip Collett

Govan Mbeki Mathematics Development Unit

Nelson Mandela Metropolitan University

Port Elizabeth, RSA

philip.collett@nmmu.ac.za

INTRODUCTION

Dynamic Visualisations is based on a series of 42 Applets which were created in GeoGebra to help you visualise a range of key concepts in CAPS Mathematics, particularly in the areas of Functions and Euclidean Geometry.

Each applet has been designed to illustrate some key concepts in a practical way. Each applets should be used in conjunction with a worksheet which will lead you through an investigation of the concepts and pose important questions.

You should complete the worksheets to the best of your ability. Often the applet can be used to check algebraic working or geometrical facts. A thorough completion of the worksheets with the aid of the applets should lead to a deeper understanding of the material.

Use the worksheets as an opportunity to learn the language of Mathematics. The vocabulary icon appears on many of the sheets and this signals that you should try to link mathematical terms to the general English meaning of the words and in particular, what the words mean to you. This is an opportunity to learn important new words and to express yourself.

Check your work with your teacher a teacher or a friend who understands these topics well. At a glance they will be able to see if you are on the right track and be able to guide you where you need help.

Take pride in your workbook. You can do extra sketches, calculations and investigations on the blank pages as well as make summary notes on what you have learned. Your workbook could become a useful revision resource for you.

The applets which form part of **Dynamic Visualisations** are available for android tablets and also on Windows for laptop and desktop computers. If you do not have a tablet try to find the applets on a desktop at your school. If you can access the internet, you can search for the applets on GeoGebraTube. (https://www.geogebratube.org/)

Next steps? When you have completed the workbook you could consider learning how to make your own constructions and functions on GeoGebra. You may be able to find GeoGebra already installed on a computer at school or download it free at home by going to www.geogebra.org. Why not try it out with a friend or start a GeoGebra club at your school. You will have fun and learn a lot of Mathematics.

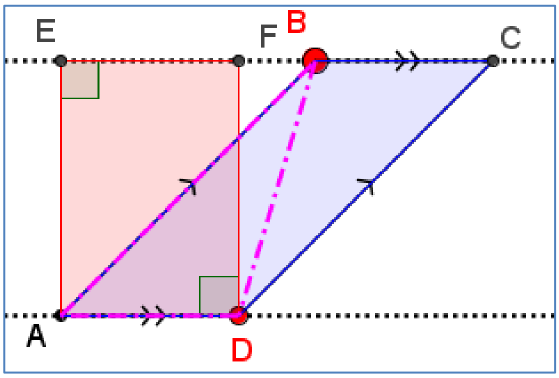
Key: Using the Icons in this Booklet

Open:	Find and open a GeoGebra Applet
Activity:	Instructions for using the GeoGebra Applet
Note:	Pay careful attention to the indicated details. They are keys to completing the worksheet.
Calculate:	Use your calculator to determine or check a result.
Complete:	Complete the sentence, or do a problem on paper.
Conjecture:	Your proposal, ‘hunch’ or educated guess of how something works.
Question:	Pose your own question. Make up a problem. Query a result.
Properties:	Make a list of properties or features; classify
Vocabulary:	Explain the meaning of a term in your own words.
Connections:	Write down an association: another word or concept that springs to mind.
Share:	Share your work with a friend, teacher or parent.

Contents

Areas of Quadrilaterals and Triangles

(Open the Applet: areas of quads and triangles)



Activity: Drag the vertices B or D to change the shape and position of the Parallelogram. Drag point B so that it co-incides with point F.

Note: Diagonal BD bisects the area of Parallelogram ABCD:

- The lengths of sides
- The size of angles
- The lengths of the diagonals
- The angle of intersection of the diagonals

Complete:

Triangles ABD and CDB are ________________ (reason: ___________)

What is the relationship between the Area of triangle ABD and parallelogram ABCD? :

____

Write down a formula for the area of triangle ABD.

____

Write down a formulas for the area of parallelogram ABCD.

____

What is the relationship between the Area of rectangle AEFD and parallelogram ABCD? :

____

____

Theorem:

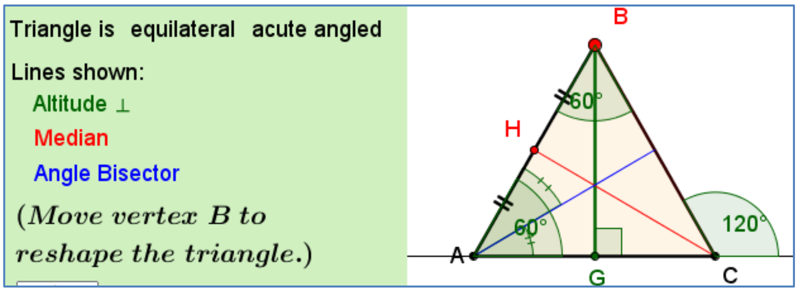
Parallelograms with the same base and the same vertical height have ____________ ____________________.

Vocabulary: congruent:

____

Triangle Properties

(Open the Applet: triangle properties)



Activity: Drag the vertex B to change the shape of triangle ABC.

Note: The following lines: altitude, median and angle bisector. (only one of each of the lines is drawn. Obviously the lines can be drawn in other positions, through the other vertices.)

Note description of the triangle in the panel on the top left of the applet.

Complete: List as many different types of triangles as you have been able to see on the applet by changing the shape of the triangle. (e.g. right-angled isosceles)

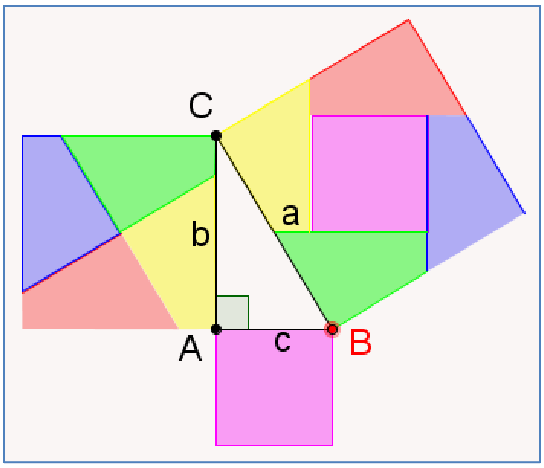
Investigate: For which kinds of triangles do the three lines (altitude, median and angle bisector) cut each other at the same point.

Vocabulary:

concurrent: ____________________

Theorem of Pythagoras

(Open the Applet: pythagoras)



Activity: Drag the vertex B to change the shape of triangle ABC.

Note: The coloured squares and quadrilaterals and how they fit together.

Complete: **What do you notice about the arrangement of the coloured shapes? Describe in your own words:**

____

____

Investigate: As you change the shape of the triangle does the relationship between the shapes change? Explain.

____

____

Theorem: State the theorem of Pythagoras in detail.

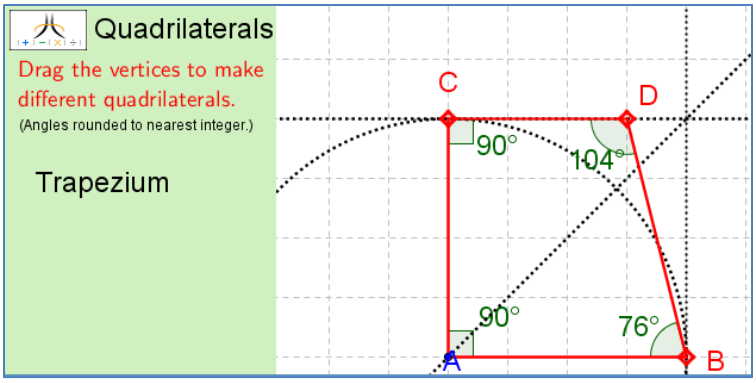
____

____

Vocabulary:

hypotenuse: ____

Classification of Quadrilaterals



(Open the Applet: quadrilaterals)

Activity: Drag the vertices
B, C or D to change the shape of the Quadrilateral

Note: Take careful note of the classification of the Quadrilateral. Notes also

- The lengths of sides (not marked – you will have to estimate)
- The size of angles

Definition:

____

____

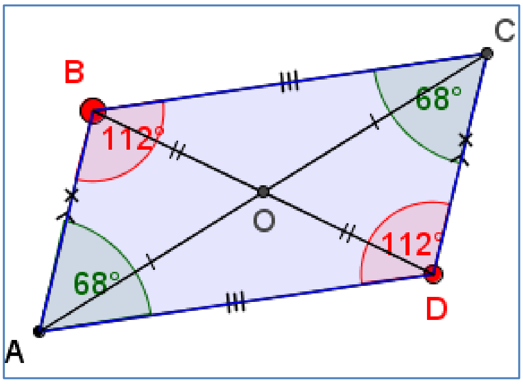
Properties: Opposite sides: ____________________________________

Opposite angles: ____________________________________

Diagonals cut at ____________________________________

Vocabulary:

: ____



Properties of a Parallelogram

(Open the Applet: Parallelogram properties)

Activity: Drag the vertices B or D to change the shape of the Parallelogram

Note: Take careful note of the properties of the Parallelogram:

- The lengths of sides
- The size of angles
- The lengths of the diagonals
- The angle of intersection of the diagonals

Complete:

Definition: A parallelogram is a quadrilateral with ________________________________.

Write your own alternative definition:

____

____

Properties: Opposite sides: ____

Opposite angles: ____

Diagonals cut at ____

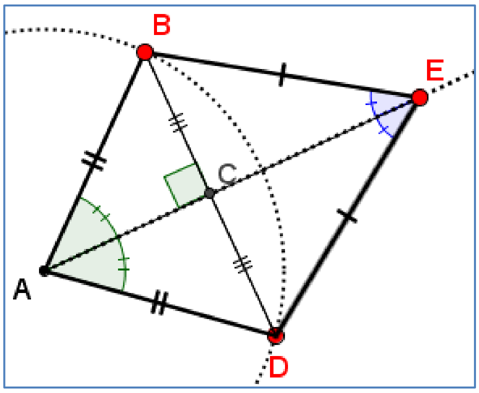
Diagonals ____________________ each other

Vocabulary:

parallelopiped: ____

Properties of a Kite

(Open the Applet: kite properties)



Activity: Drag the vertices B, C or D to change the shape of the kite

Note: Take careful note of the properties of the kite:

- The lengths of sides
- The size of angles
- The lengths of the diagonals
- The angle of intersection of the diagonals

Complete:

Definition: A kite is a quadrilateral with ________ sides equal.

Write your own alternative definition:

____

____

Properties: Opposite angles: ____

Diagonals cut at ____

One diagonal ___________________ the other

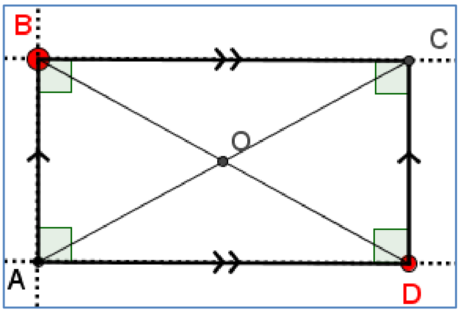
Vocabulary:

Convex: ____

Concave: ____

Connect: A kite which is a parallelogram is a ___________

Properties of a Rectangle



(Open the Applet: rectangle properties)

Activity: Drag the vertices B or D to change the shape of the rectangle

Note: Take careful note of the properties of the rectangle:

- The lengths of sides
- The size of angles
- The lengths of the diagonals
- The angle of intersection of the diagonals

Complete:

Definition: A rectangle is a parallelogram with ________________________________.

Write your own alternative definition if possible:

____

____

Properties: Opposite sides: ____

Diagonals cut at ____

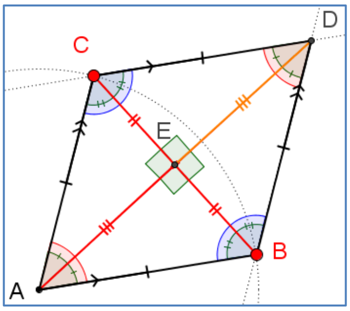
Diagonals ____________________ each other

Vocabulary:

Oblong: ____

Connections:

Properties of a Rhombus



(Open the Applet: Rhombus properties)

Activity: Drag the vertices B or D to change the shape of the Rhombus

Note: Take careful note of the properties of the Rhombus:

- The lengths of sides
- The size of angles
- The lengths of the diagonals
- The angle of intersection of the diagonals

Complete:

Definition: A rhombus is a __________________ with _____________________________.

Write your own alternative definition:

____

____

Properties: Opposite sides: ____

Opposite angles: ____

Diagonals cut at ____

Diagonals __________________ each other

Vocabulary:

Rhomboid: ____

Connections:

Linear Function

(Open the Applet: linear function)

Activity: (1) Change the value of variable ***c*** by dragging the slider.

Note: The position of the y-intercept

Activity: (2) Change the value of variable ***m*** by dragging the slider.

Note: The slope and orientation of the graph (/ or \ )

Calculate:

Find the root of the equation f(x)=0 where m=-1 and c=4.

Show your working: ___________________________________.

____

____

Check your work by entering the values on the Applet.

Conjecture:

The equation has no real roots if m _____ and c __________

Vocabulary:

gradient: ____

Connections:

Quadratic Function ($f(x) = ax^2 + c$)

(Open the Applet: Quadratic Function)

Activity: (1) Change the value of variable c by dragging the slider.

Note: The position of the y-intercept

Activity: (2) Change the value of variable a by dragging the slider.

Note: The width and orientation of the graph (∩ or U)

Complete:

Find the roots of the equation f(x)=0 where a=-1 and c=4.

Show your working: ________________________________.

____

____

Check your work by entering the values on the Applet.

Complete:

The equation has no real roots if a _____ and c _________

Or if a _____ and c _________

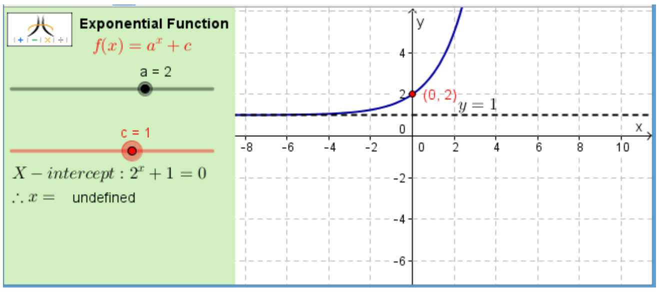
Vocabulary:

Parabola: ____

Exponential Function

($f(x) = a^x + c$)

(Open the Applet: exponential function 10)



Activity: (1) Change the value of variable c by dragging the slider.

Note: The position of the y-intercept

Activity: (2) Change the value of variable a by dragging the slider.

Note: The steepness and orientation of the graph (⮥ or ↪)

Find the root of the equation f(x)=0 where a=2 and c= - 4.

Show your working: ________________________________.

____

The Y-intercept is the point (0, ____)

Check your work by entering the values on the Applet.

Complete:

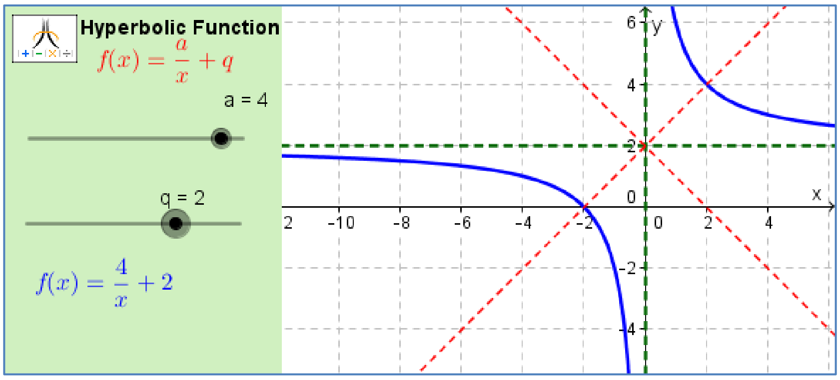
The graph of f(x) does not cross the X-axis if c _________

If 0<a<1 then ____

Vocabulary:

Assymptote: ____

Hyperbolic Function ($f(x)=\frac{a}{x}+q$)



(Open the Applet: hyperbolic function 10)

Activity: (1) Change the value of variable **a** by dragging the slider.

Note: The shape and orientation of the graph

Activity: (2) Change the value of variable **q** by dragging the slider.

Note: The vertical position of the graph.

Calculate:

Find the zeroes of the equation f(x)=0 where a= -2 and q=2.

Show your working: ________________________________.

____

____

Check your work by entering the values on the Applet.

By Inspection

Write down the equations of the axes of symmetry: ________________________________.

____

____

Check your work by entering the values on the Applet.

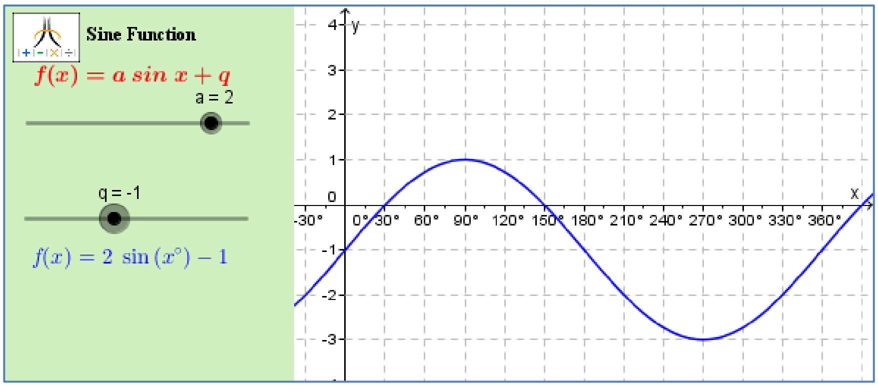
Conjecture:

The function has no zeroes if the value of q is _______, otherwise the function has ___ zero which has the value _______ (express in terms of a and q).

Vocabulary: Symmetrical: ____

Sine Function (G10)

($f(x) = a \cdot \sin(x) + q$)



(Open the Applet: sine function 10)

Activity: (1) Change the value of variable **a** by dragging the slider.

Note: The change in vertical height of the graph

Activity: (2) Change the value of variable **q** by dragging the slider.

Note: The vertical position of the graph.

Calculate:

The y – intercept of the function defined by f(x)=2sin(x˚) - 1 Show your working:

____

____

Check your work by entering the values on the Applet (a=2; q= -1).

Calculate:

Find a solution (or two) for the equation 2sin(x˚) - 1=0. Show your working:

____

____

Check your work by entering the values on the Applet.(a=2; q= - 1)

Conjectures:

As the value of a increases , the amplitude of the function defined by y=a sin x ______________(increases/decreases/stays the same)

The graph of the function defined by y=sin (x)+q can be shifted down ___________________________(increasing/decreasing) the value of q.

Vocabulary: Periodic: ____

Cosine Function G10 ($f(x) = a \cdot \cos(x) + q$)

(Open the Applet: cosine function 10)

Activity: (1) Change the value of variable **a** by dragging the slider.

Note: The change in vertical height of the graph

Activity: (2) Change the value of variable **q** by dragging the slider.

Note: The vertical position of the graph.

Calculate:

The y – intercept of the function defined by f(x)=2cos(x˚) - 1 Show your working:

____

____

Check your work by entering the values on the Applet (a=2; q= -1).

Calculate:

Find a solution (or two) for the equation 2cos(x˚) - 1=0. Show your working:

____

____

Check your work by entering the values on the Applet.(a=2; q= - 1)

Conjecture:

The function defined by y=a cos (x) +q has no zeroes if __________________________

____

Vocabulary:

Co-function:____

Tangent Function G10 ($f(x)=a\cdot\tan(x)+q$)

(Open the Applet: Tangent Function 10)

Activity:(1) Change the value of variable **a** by dragging the slider.

Note: The change in the steepness of the graph.

Activity: (2) Change the value of variable **k** by dragging the slider.

Note: (Period) The interval over which the graph does one cycle before repeating itself.

Activity: (3) Change the value of variable **p** by dragging the slider.

Note: The horizontal position of the graph.

Activity: (4) Change the value of variable **q** by dragging the slider.

Note: The vertical position of the graph.

Calculate:

The y – intercept of the function defined by f(x)= 2 tan2(x) +3. Show your working:

Check your work by entering the values on the Applet (a= 2; k=2, p= 0˚; q=3).

Calculate:

Find a solution (or more) for the equation 3 tan 2(x-30˚) - 3 =0. Show your working:

Check your work by entering the values on the Applet.(a=3; k=2, p= - 30˚; q= -3)

Conjecture:

The function defined by y= tan 3(x-p) has no zeroes if p has the value(s)

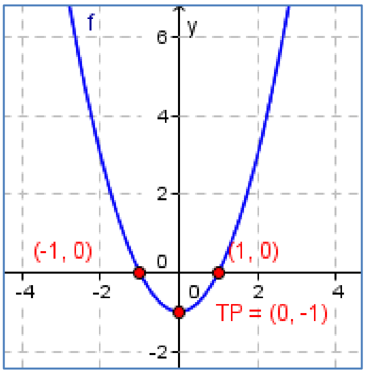
____

Vocabulary:

tangential:____

Connections:

Quadratic Function ($f(x) = ax^2 + bx + c$)



(Open the Applet: quadratic function 11)

Activity: (1) Change the value of variable c by dragging the slider.

Note: The position of the y-intercept

Activity: (2) Change the value of variable a by dragging the slider.

Note: The width and orientation of the graph (∩ or U)

Activity: (3) Change the value of variable b by dragging the slider.

Note: The effect on the graph is ____________________________________

Complete:

Find the roots of the equation f(x)=0 where a=-1, b=3 and c=4.

Show your working: _____________________________.

____

____

Check your work by entering the values on the Applet.

Investigation: **Choose sets 4 sets of values so that the equation**
$f(x) = ax^2 + bx + c = 0$ as no real roots. (In each case the graph must entirely miss the x-axis.)

Fill out the following table:

Graph no.	a	b	c	$b^2 - 4ac$
1				
2				
3				
4				

Conjecture:

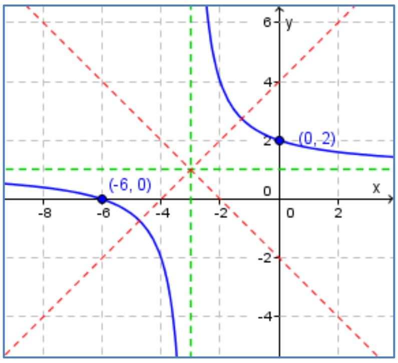
The equation has no real roots if ______________________________

The axis of symmetry has the equation: ___________________________

Vocabulary:

Symmetrical: ____

Hyperbolic Function ($f(x)=\dfrac{a}{x-p}+q$)



(Open the Applet: Hyperbolic Function)

Activity: (1) Change the value of variable **a** by dragging the slider.

Note: The shape and orientation of the graph

Activity: (2) Change the value of variable **p** by dragging the slider.

Note: The horizontal position of the graph.

Activity: (3) Change the value of variable **q** by dragging the slider.

Note: The vertical position of the graph.

Calculate:

Find the zeroes of the equation f(x)=0 where a= -1, p=1 and q=1.

Show your working: __________________________________.

____

____

Check your work by entering the values on the Applet.

Calculate:

The y – intercept of the function defined by f(x)=0 where a= -1, p=1 and q=1.

Show your working: __________________________________.

____

____

Check your work by entering the values on the Applet.

Conjecture:

The equation has no real roots if ______________________________

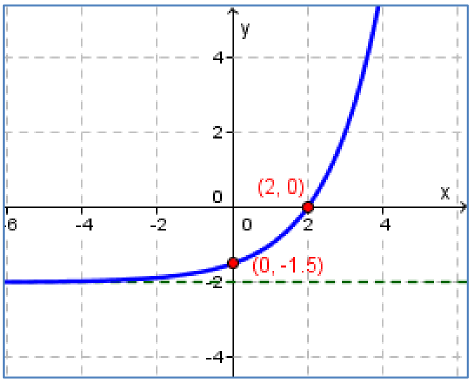
The axis of symmetry has the equation: ___________________________

Vocabulary:

Symmetrical: ____

Connections:

Exponential Function ($f(x)=a\cdot b^{x-p}+q$)



(Open the Applet: Exponential Function)

Activity: (1) Change the value of variable **a** by dragging the slider.

Note: The steepness and orientation of the graph

Activity: (2) Change the value of variable **b** by dragging the slider.

Note: The steepness and orientation of the graph

Activity: (3) Change the value of variable **p** by dragging the slider.

Note: The horizontal position of the graph.

Activity: (4) Change the value of variable **q** by dragging the slider.

Note: The vertical position of the graph.

Calculate:

The y – intercept of the function defined by y= f(x) where a= 1, b = 2, p =1 and q = -2.

Show your working: ________________________________.

____

____

Check your work by entering the values on the Applet.

Challenge:

Find the zeroes of the equation f(x)=0 where a= -1, p=1 and q=1. (Hint: use logs)

Show your working: ________________________________.

____

____

Check your work by entering the values on the Applet.

Conjectures:

The equation of the horizontal asymptote is:______________________________

For these functions to be defined the value of b must be ___________________________

Vocabulary:

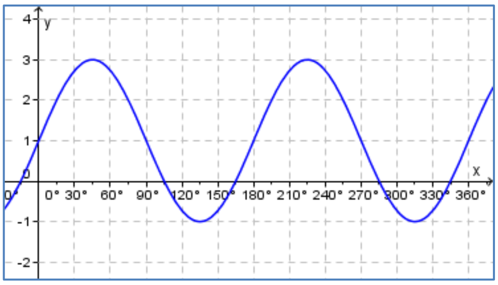
Assymetrical:____

Connections: Compound Growth

Write down a formula:

Sine Function

($f(x) = a \cdot \sin(k(x - p)) + q$)



(Open the Applet: Sine Function)

Activity: (1) Change the value of variable **a** by dragging the slider.

Note: The change in vertical height of the graph

Activity: (2) Change the value of variable **k** by dragging the slider.

Note: The interval over which the graph does one cycle before repeating itself.
(Period)

Activity: (3) Change the value of variable **p** by dragging the slider.

Note: The horizontal position of the graph.

Activity: (4) Change the value of variable **q** by dragging the slider.

Note: The vertical position of the graph.

Calculate:

The y – intercept of the function defined by f(x)=2sin(x+30˚)+1 Show your working:

___

___

Check your work by entering the values on the Applet (a=2; k=1, p= -30˚; q=1).

Calculate:

Find a solution (or two) for the equation 2sin(x+30˚)+1=0. Show your working:

___

___

Check your work by entering the values on the Applet.(a=2; k=1, p= -30˚; q=1)

Conjectures:

As the value of k increases , the period of the function defined by y=sin kx ______________

(increases/decreases/stays the same/is unpredictable)

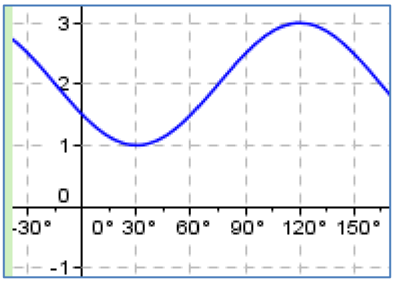
The graph of the function defined by y=sin (x-p) can be shifted to the right by ___________________________(increasing/decreasing) the value of p.

Vocabulary:

Periodic:____

Connections: Waves

Cosine Function ($f(x) = a \cdot \cos(k(x-p)) + q$)



(Open the Applet: Cosine Function)

Activity: (1) Change the value of variable **a** by dragging the slider.

Note: The change in vertical height of the graph

Activity: (2) Change the value of variable **k** by dragging the slider.

Note: The interval over which the graph does one cycle before repeating itself. (Period)

Activity: (3) Change the value of variable **p** by dragging the slider.

Note: The horizontal position of the graph.

Activity: (4) Change the value of variable **q** by dragging the slider.

Note: The vertical position of the graph.

Calculate:

The y – intercept of the function defined by f(x)= -cos2(x-30˚)+2. Show your working:

___

___

Check your work by entering the values on the Applet (a= -1; k=2, p= 30˚; q=2).

Calculate:

Find a solution (or two) for the equation cos 2(x-30˚) =0. Show your working:

___

___

Check your work by entering the values on the Applet.(a=1; k=2, p= 30˚; q=0)

Conjecture:

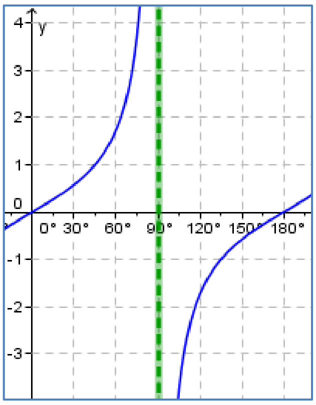
The function defined by y=a cos k(x-p) +q has no zeroes if ____________________________

____

Vocabulary:

Co-function:____

Connections: Waves



Tangent Function ($f(x) = a \cdot \tan(k(x-p)) + q$)

(Open the Applet: Tangent Function)

Activity:(1) Change the value of variable ***a*** by dragging the slider.

Note: The change in the steepness of the graph.

Activity: (2) Change the value of variable **k** by dragging the slider.

Note: (Period) The interval over which the graph does one cycle before repeating itself.

Activity: (3) Change the value of variable **p** by dragging the slider.

Note: The horizontal position of the graph.

Activity: (4) Change the value of variable **q** by dragging the slider.

Note: The vertical position of the graph.

Calculate:

The y – intercept of the function defined by f(x)= 2 tan2(x) +3. Show your working:

____

____

Check your work by entering the values on the Applet (a= 2; k=2, p= 0˚; q=3).

Calculate:

Find a solution (or more) for the equation 3 tan 2(x-30˚) - 3 =0. Show your working:

____

____

Check your work by entering the values on the Applet.(a=3; k=2, p= - 30˚; q= -3)

Conjecture:

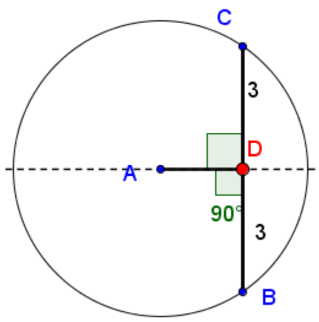
The function defined by y= tan 3(x-p) has no zeroes if p has the value(s) ___________________________

Vocabulary:

tangential:____

Connections:

Centre -Chord Theorems



(Open the Applet: centre-chord)

Activity: Drag point D to investigate the relationship between the centre of the circle and a chord.

Note: Take careful note of:

- The lengths of the two segments which make up the chord.
- The size of angles
- The position of the perpendicular line.

Complete the following Theorems:

1. A line from the centre of the circle which is perpendicular to the chord ____________

____

2. A line from the centre to the midpoint of the chord is

   ____

3. The perpendicular bisector of the chord ______________________________ the centre of the circle .

Statements 1 and 2 above can be proved by using ______________triangles.

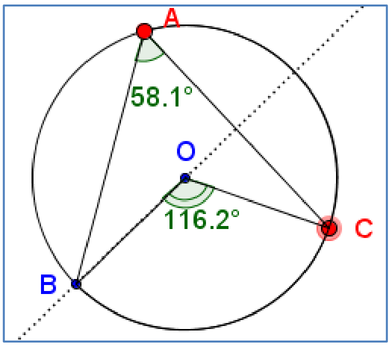
Statement 3 can be proved by assuming that the perpendicular bisector does not pass through the centre of the circle and then showing that this assumption leads to a contradiction. Try it for yourself (start with a sketch):

Vocabulary:

Contradiction: ____

Angles at the centre and at the circumference.

(Open the Applet: angle at centre)



Activity: Drag point C (or A) to investigate the relationship between the angle at the centre of the circle and the angle at the circumference.

Note:

- The size of angles
- The angles when BC becomes a diameter.

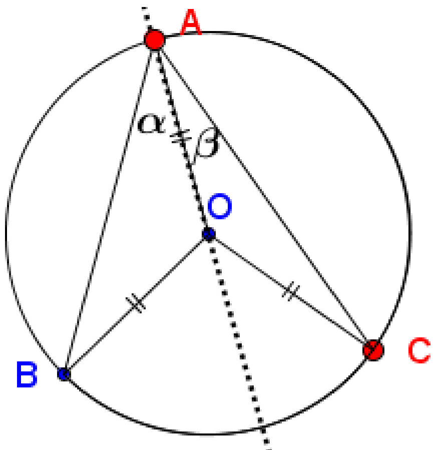
Complete the following Theorems:

1. The angle subtended by a chord at the centre of the circle is

____

2. The angle in a semicircle is ____

Statement 1 can be proved by drawing a line through the centre and the point on the circumference (OA) and using known facts about isosceles triangles and the exterior angle of a triangle.



Fill in the missing angles and try the proof for yourself:

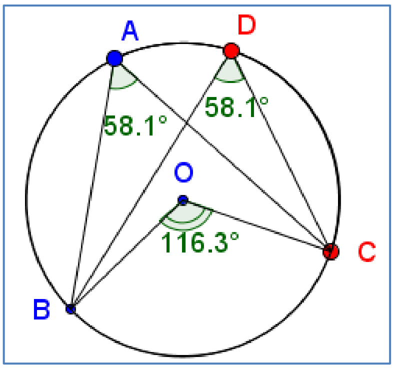
Required to prove: ______________

Construction: ____________________

Proof:

Vocabulary: Reflex angle: ____

Angles in the same segment.



(Open the Applet: angle in segment)

Activity: Drag point D to investigate what happens to the angle at the circumference. Drag point C to change the length of the chord and then move D around.

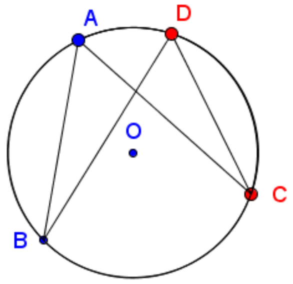
Note: The size of angles

Complete the following Theorem:

1. Angles subtended by a chord at the circumference of a circle, on the same side of the chord, are __________.

Alternatively: Angles in the same segment are __________.

Try to **prove the theorem** for yourself: (Hint: use the angle at the centre)



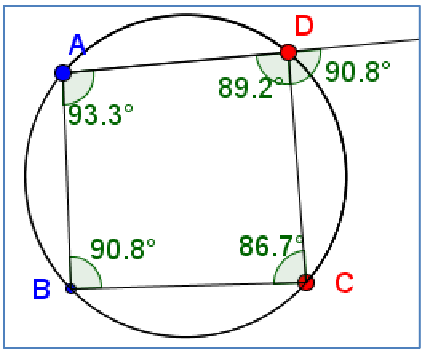
Required to prove: _____________

Construction: ____________________

Proof:

Vocabulary: segment: ____

Angles of a cyclic quadrilateral.



(Open the Applet: angles of cyclic quad)

Activity: Drag points D (or C) to investigate the angles.

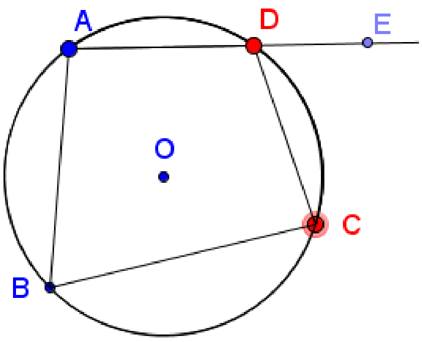
Note: The size of angles, focusing on the opposite angles.

Complete the following Theorem:

1. The opposite angles of a cyclic quadrilateral are ______________________.
2. The exterior angle of a cyclic quadrilateral is ______________________________

   ____.

Try to **prove these theorems** for yourself: (Hint: use the angle at the centre)



Required to prove: _____________

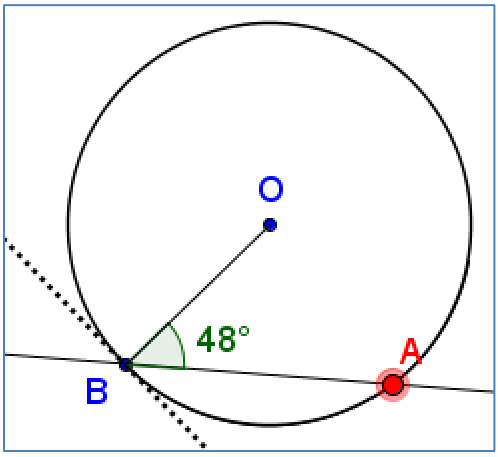
Construction: ___________________

Proof:

Share your proof with your teacher or someone who knows circle geometry – or check the proofs in a textbook.

Angle between a radius and a tangent.

(Open the Applet: tangent radius)



Activity: Drag point **A** to investigate the angle.

Note: The size of angle as point A approaches B, the point of contact of the tangent indicated by the dotted line.

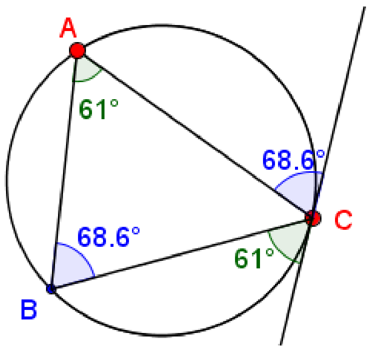
Complete the following Theorem:

A tangent to a circle is ________________ to the radius at the point of __________.

Vocabulary:

Secant: ____

Chord and a tangent.



(Open the Applet: tangent chord)

Activity: Drag point C to investigate the angles.

Note: Compare the sizes of angles

Complete the following Theorem:

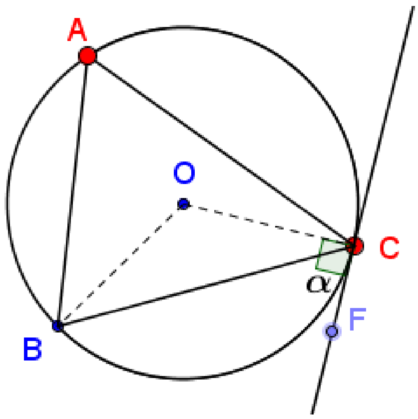
The angle between a chord and the tangent to a circle (at one end of the chord) is equal to ____.

Prove this theorem for yourself: (Hint: use the angle at the centre and the fact that the tangent is perpendicular to the radius. Label all the angles in terms of α)

Required to prove: _____________

Construction: ___________________

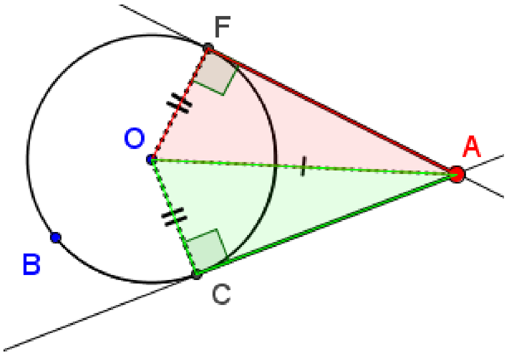
Proof:



Share your proof with your teacher or someone who knows circle geometry – or check the proofs in a textbook.

Tangents from a point.

(Open the Applet: tangents from a point)



Activity: Drag point A to change the length of the tangents and the shape of the triangles.

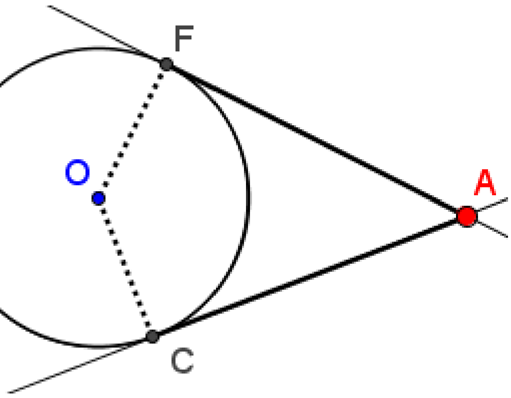
Note: Compare the lengths of the line segments.

Compare the shape of the triangles.

Complete the following Theorem:

The length of tangents to a circle from a common point are ______________________.

Prove this theorem for yourself: (Hint: use congruent triangles)



Given: ______________

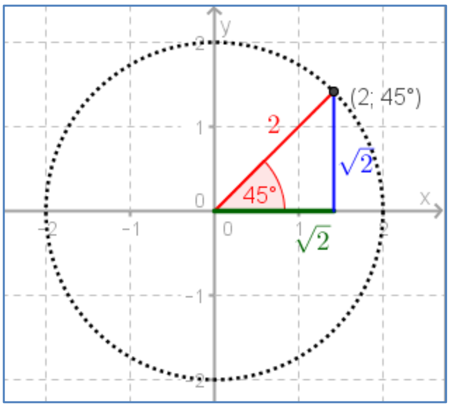
Required to prove: ______________

Construction: ____________________

Proof:

In Δ's *OFA* and *OCA*,

Share your proof with your teacher or someone who knows circle geometry – or check the proofs in a textbook.



Trigonometric ratios and identities

(Open the Applet: trig ratios and identities)

Activity: Change the value of the angle θ(theta) by dragging the point on the 'slider'.

Note: The lengths of the opposite, adjacent sides and the hypotenuse.

The relationship between the values of sinθ, cosθ and tanθ.

Calculate: by changing the angle in the applet and recording the lengths of the sides of the triangle, calculate the following:

Cos 60˚= ---- =

Sin135˚= ---- =

Tan 285˚= ---- =

Sec240˚= ---- =

Complete the following : $\frac{\sin\theta}{\cos\theta} =$

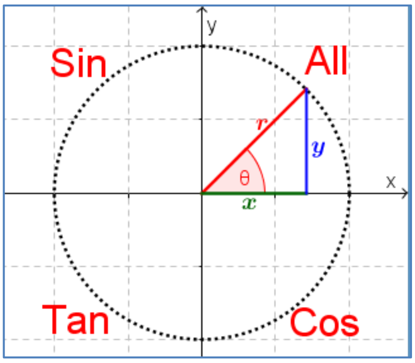
Investigate

θ	Sin θ	Cos θ	$\text{Sin}^2\,\theta$	$\text{Cos}^2\,\theta$	$\text{Sin}^2\,\theta + \text{Cos}^2\,\theta$
30˚					
45˚					
60˚					
90˚					

Conjecture: $\sin^2\theta + \cos^2\theta =$

Try to prove this: (Hint: use the definitions of Sin θ and Cos θ and do the algebra.)

Reduction Formulae



(Open the Applet: reduction formulae)

Activity: Change the quadrant by dragging the point on the 'slider'.

Note: The signs of the opposite, adjacent sides and the hypotenuse.

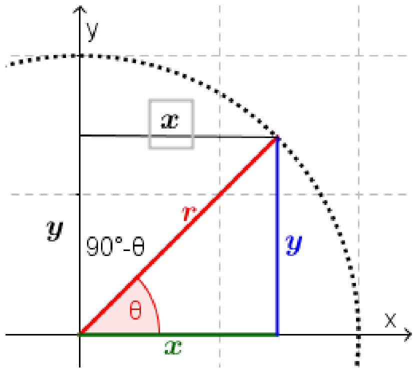
Investigate: Complete the following table by writing down the sign of each trigonometric ratio (+ or -)

quadrant	interval	sin θ	cos θ	tan θ
1	$0^\circ \leq \theta \leq 90^\circ$			
2				
3				
4				

Conjecture: $\sin(90° - \theta) =$ $\qquad\qquad =$

$\cos(90° - \theta) =$ $\qquad\qquad =$

(Hint: use the diagram to the right.)



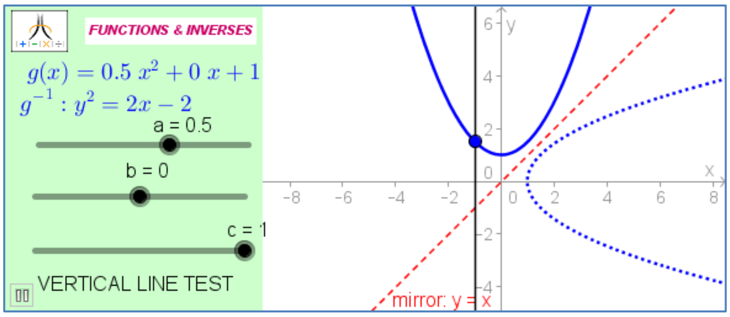
Complete the following reduction formulae by using the applet:

$\sin(180° - \theta) =$

$\cos(180° + \theta) =$

$\tan(360° - \theta) =$

Inverse Functions



(Open the Applet: inverse functions)

Activity: Change the shape of the parabola by dragging the 'sliders' **a**, **b** and **c**.

Note: The shape of the inverse relation (dotted) in relation to the mirror line, y=x.

Investigate: To help you do this, work on the drawing above. Choose a point on the function. Then draw a line through this point perpendicular to the line y=x. Continue this line till it cuts the inverse. Measure the line segments which you have drawn. What do you notice?

Watch the animation. Notice how many times vertical line (black) cuts the graphs at any one value of x:

Parabola: The vertical line cuts a maximum of ____________ times. Inverse: The vertical line cuts a maximum of ____________ times.

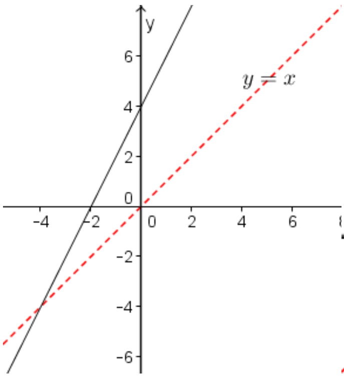
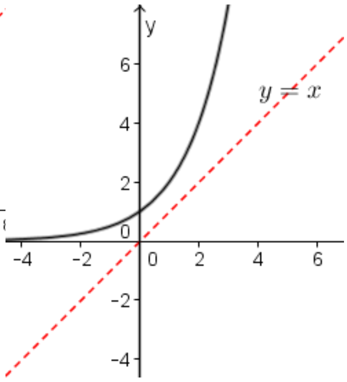
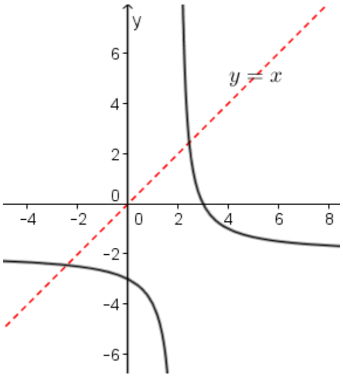
Conjecture: The ______________ is a function and the __________ is not.

Vocabulary:

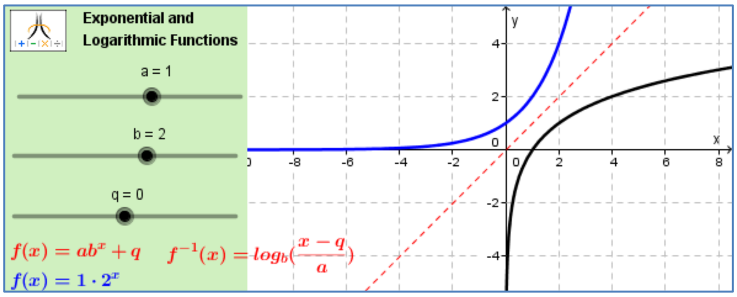
Symmetrical: The graphs of the function and its inverse forms a ____________ figure and the line y=x is the axis of ____________________

Drawing inverse graphs:

On the figures provided draw the graphs of the inverses of the straight line, the exponential function and the hyperbola. Use the idea of symmetry.


y
$y = x$

y
$y = x$

y
$y = x$

Exponential and Logarithmic Functions



(Open the Applet: exponent and log functions)

Activity: Change the shape of the curves by dragging the 'sliders' a, b and q.

Note: How the exponential functionand the logarithmic function together form a symmetrical figure with a line of symmetry of y=x.

Complete: The inverse of an exponential function is a ____________ function.

Determine: The inverse of the function defined by $y = 2 \cdot 3^x - 2$ by algebraic means.

Check your answer by using the applet with a=2, b=3 and q= -2.

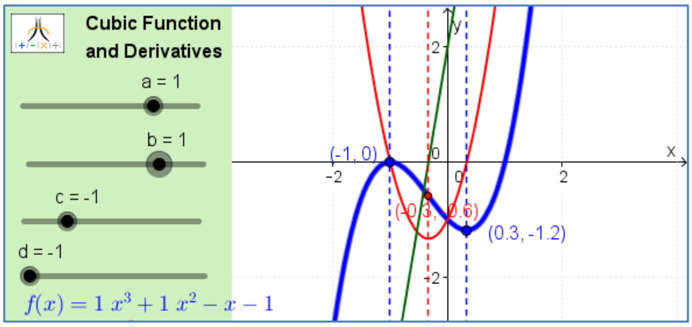
Determine: The inverse of the function defined by $y = 2 \cdot \left(\frac{1}{2}\right)^x$ by algebraic means.

Check your answer by using the applet with a=2, b=0.5 and q= 0.

Complete: If the horizontal asymptote of an exponential function is y=2 then the vertical asymptote of the inverse function is ____________ .

Cubic Functions

(Open the Applet: cubic functions)



Activity: Change the shape of the curves by dragging the 'sliders' a, b and c and d.

Note: How the shape of the cubic function changes as each parameter changes.

Exercise: Consider the function defined by $f(x) = x^3 + x^2 - x - 1$

1. Use the factor theorem to find factors of f(x) and hence find the roots of the equation $x^3 + x^2 - x - 1 = 0$

2. Use derivatives to find the turning points and inflection point of the function.

Use the applet to check your working. Set a=1, b=1, c= -1 and d= -1. (Note that the accuracy of the coordinates in the applet is to 1 decimal place.)

What do you notice about the x-value of the coordinates of the turning points and the zeros of the first derivative? ____

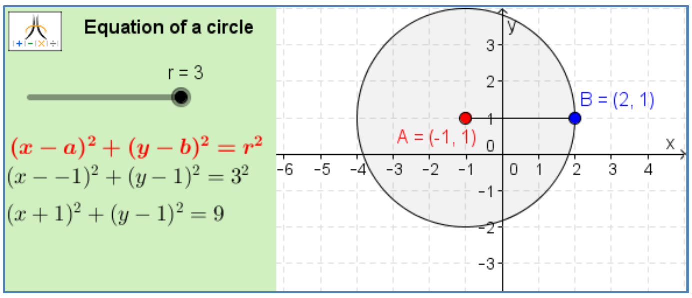
What do you notice about the turning point of the first derivative and the zero of the second derivative?____

Does the cubic function always have a local minimum and maximum? Write down the defining equation of a function which does not seem to have these extreme points. (Use the applet to explore.)

f(x)= ____x^3 ____ x^2 _____x ______

<u>Worksheet</u>: Equation of a circle

(Open the Applet: circle equation)



Activity: change the centre of the circle by moving point A (the red dot). Try changing the value of r (the radius) to make the circle bigger or smaller.

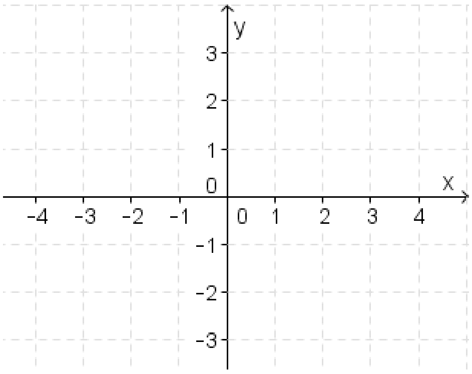
Note: How the changes in the equation of the circle.

Exercise: Use the applet to create a circle with the equation $(x-1)^2 + (y-1)^2 = 4$.

Write down the values for this circle: a =___, b =____, and r =_____

Change the equation $x^2 + 4x + y^2 - 2y + 1 = 0$ into the form $(x-a)^2 + (y-b)^2 = r^2$. You will need to use the completion of the square method.

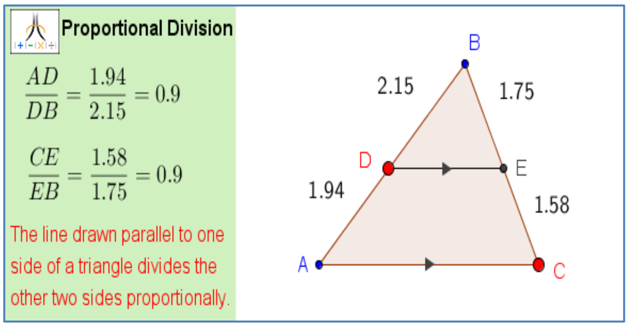
With the help of the applet, sketch this circle showing clearly the centre, the radius and the points at which the circle cuts the axes.



Try to calculate the intercepts with the axes by setting x=0 (y-intercept) and y=0 (x-intercept). Check your results with the applet.

Proportional Division

(Open the Applet: proportional division)



Activity: Drag point D so that it cuts AB in a ratio. (you can also drag point C to change the shape of the triangle.)

Note: The ratios $\frac{AD}{DB}$ and $\frac{CE}{EB}$

Complete the following : $\frac{AD}{DB} =$ $\qquad\qquad$ $\frac{CE}{EB} =$

$\frac{DB}{BA} =$ $\qquad\qquad$ $\frac{BE}{BC} =$

$\frac{DB}{BE} =$ $\qquad\qquad$ $\frac{BA}{BC} =$

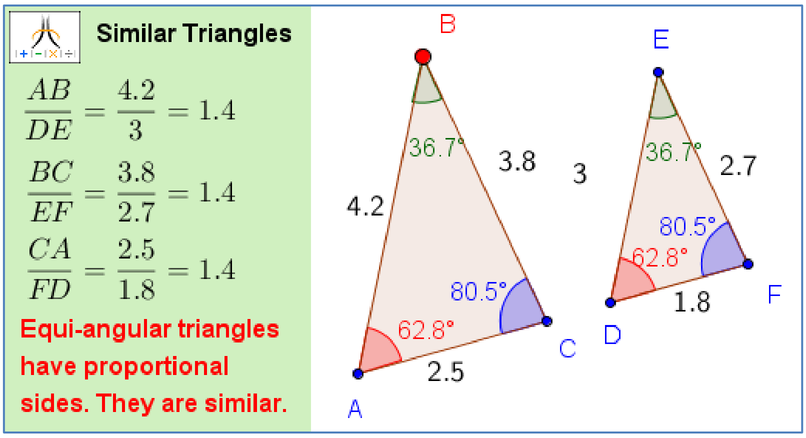
Conclusions: (write down the ratios which are equal)

Complete the following Theorem:

A line drawn parallel to one side of a triangle divides the other two sides in the same ____________________.

Similarity

(Open the Applet: similarity)



Activity: Drag point B to change the shape of triangle ABC.

Note: the shapes of the two triangles and the lengths of the sides.

Complete the following : $\frac{AB}{DE} =$ $\frac{BC}{EF} =$ $\frac{AC}{DF} =$

$\frac{AB}{AC} =$ $\frac{DE}{DF} =$

$\frac{AB}{BC} =$ $\frac{DE}{EF} =$

Conclusions: (write down the ratios which are equal)

Complete the following Theorem:

Triangles which have equal angles have their sides in __________________ and are therefore __________________.

Conversely:

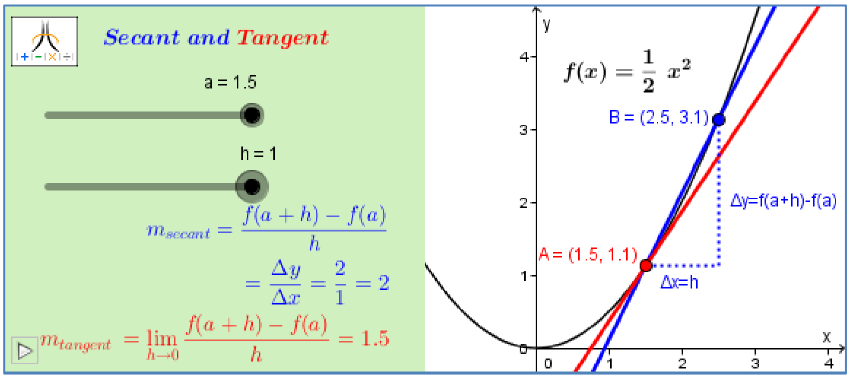
Triangles which have their __________ in proportion have ________ angles and are therefore __________________.

Congruence implies Similarity BUT Similarity does not imply Congurence.

Explain this statement in your own words or give examples to illustrate the statement:

Derivative of a function at a point

(Open the Applet: derivative definition)



Activity: Fix the value of 'slider' a at say 1.5, Click the play button. This will change the value of the variable h, through a range of values and repeat this process. (You can also manually change the value of h.)

Watch the animation.

Note: Compare the values of the slope of the secant (in blue) and the slope of the tangent.

Conclusions: The slope of the tangent is equal to the limit of the ________________________________ as the value of h approaches _________.

Complete: The derivative of a function at a point (a;f(a)) is given by the formula:

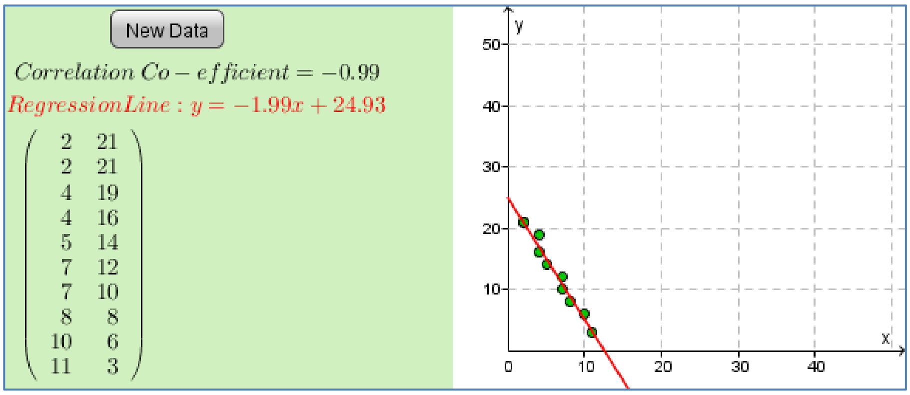
This value also gives the slope of the function.

Vocabulary: write down 2 synonyms for the following:

gradient:____

Correlation and Regression

(Open the Applet: correlation and regression)



Activity: Press the New Data button and investigate the data set, the scatter plot and the line of best fit.

Note: The values in the data set, the position of the points on the graph, the slope of the regression line its equation.

Complete: If y increases as x increases then the slope of the regression line is __________ (positive or negative)

If y decreases as x increases then the slope of the regression line is __________

The closer most points are to the regression line the closer the value of the correlation coefficient is to __________ or ___________. (fill in two numbers.)

The correlation co-efficient has the same sign as the __________ of the regression line.

Choose a New Data set and use your **calculator** to determine the following:

(a) The correlation coefficient
(b) The equation of the regression line (Line of best fit.)

Record the data set in the table below:

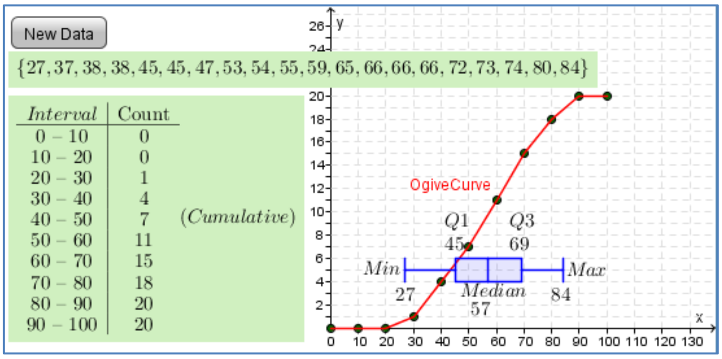
x										
y										

Note: the x- values are in the left-hand column on the applet and the y-values on the right.

Vocabulary:

bivariate:____

Summarizing Data



(Open the Applet: summarizing data)

Activity: Press the New Data button and investigate the data set, the cumulative frequency table, the ogive curve and the box plot.

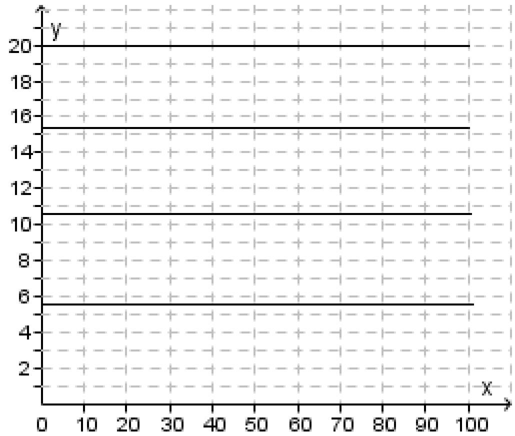
Note: The values in the data set, the shape of the ogive and the box plot.

Complete: Choose a New Data set and Record the data set in the table below:

x										

Fill in the cumulative frequency table (without referring to the applet): Plot the ogive curve:

Interval	Frequency	Cumulative frequency
0≤x<10		
10≤x<20		
20≤x<30		
30≤x<40		
40≤x<50		
50≤x<60		
60≤x<70		
70≤x<80		
80≤x<90		
90≤x≤100		



Without referring to the graphs on the applet determine the following by (1) inspection and calculation (2) reading the values off the graph:

By calculation: Min = Q1 = Median = Q3 = Max=

From the graph: Min = Q1 = Median = Q3 = Max=

(To read the values from the graph draw a vertical line down from the points of intersection of the ogive with 4 solid horizontal lines. Read the values off the x-axis where the vertical lines cut the axis.)

Explain why these sets of values are slightly different: ______________________________

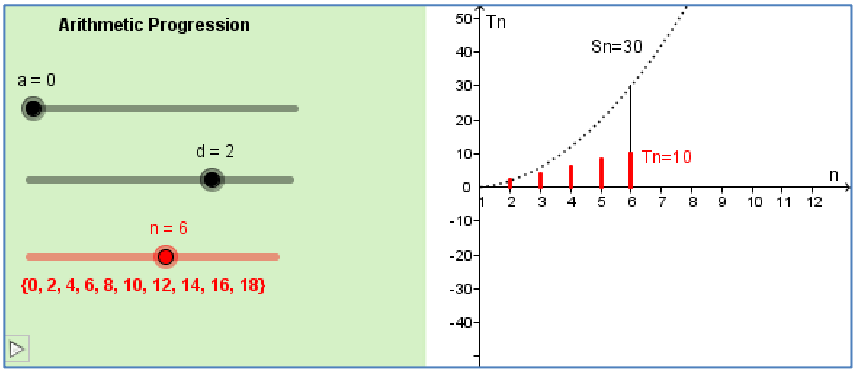
Now check your working by referring to the graphs on the applet. Take note of any differences and correct your working where you may have made mistakes.

Use the graph to find the 60^{th} percentile of the data. Mark it on the graph.

Vocabulary:
percentile:____

Arithmetic Sequences

(Open the Applet: arithmetic sequence)



Activity: Press the **Play** button (bottom right) and watch the graphical illustration.

Note: The starting value (a), the common difference (d) and the value of the terms. Also note the value of the sum (Sn) of n terms of the series.

Activity: Stop the animation (press the button in the bottom left corner) and change the values of a and d and set n=0. Press the **Play** button (bottom right) and watch the graphical illustration again with the new values.

Complete:

The line through the top of the line segments representing the value of the terms of the sequence is a __________ line, indicating a ___________ function.

The growth of Sn (the sum to n terms) can be described as a __________function.

Write down a formula for the sum in terms of n, a and d.__________________________

Change the formula so that **a** and **d** are the constants you have chosen in the applet.

Sn= ______

Now multiply the factors and express the formula in descending powers of n.

Sn= _______________________________

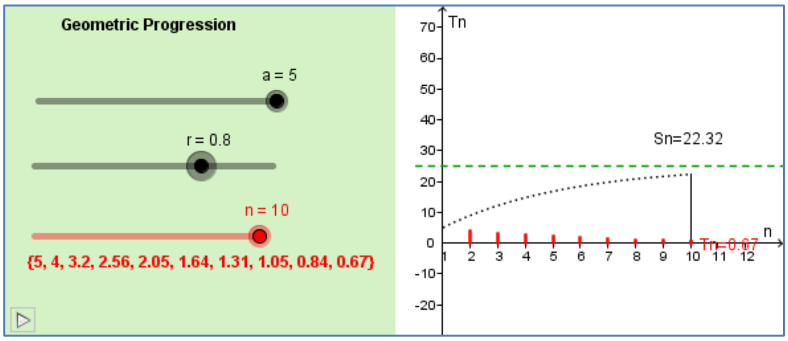
Do you recognise this formula as describing a quadratic function? ___________

Vocabulary:

series:____

descending:____

Geometric Sequences



(Open the Applet: geometric sequence)

Activity: Press the **Play** button (bottom right) and watch the graphical illustration.

Note: The starting value (**a**), the common ratio (**r**) and the value of the terms. Also note the value of the sum (**Sn**) of n terms of the series.

Activity: Stop the animation (press the button in the bottom left corner) and change the values of **a** and **r** and set n=0. Press the **Play** button (bottom right) and watch the graphical illustration again with the new values.

Complete:

The line through the top of the line segments representing the value of the terms of the sequence is a __________ line, indicating a ___________ function.

The growth of Sn (the sum to n terms) can be described as a __________function.

Write down a formula for the sum in terms of n, a and r._________________________

Change the formula so that **a** and **d** are the constants you have chosen in the applet.

Sn= ______

Now multiply the factors and express the formula in descending powers of n.

Sn= ______________________________

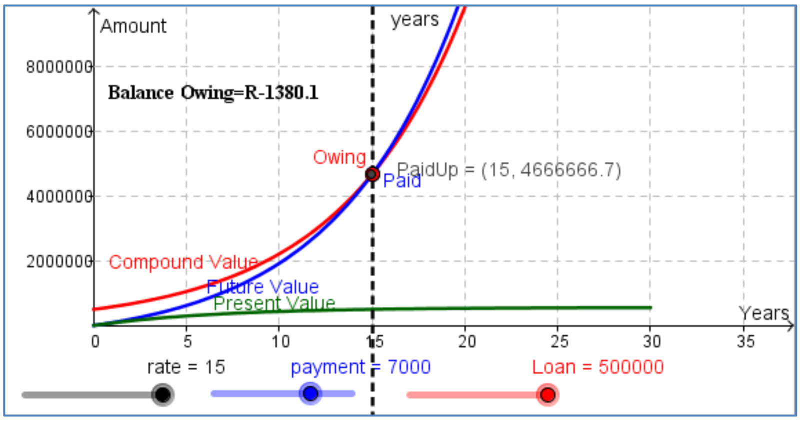
Do you recognise this formula as describing a quadratic function? ___________

Vocabulary:

convergent:____

divergent:____

Annuities



(Open the Applet: annuities)

Activity: Change the values of the three variables: **rate**, **payment** and **Loan**.

Note: The shapes of the three curves:

Compound Value (the loan with interest added, compounded monthly.

Future Value – the value of the monthly payments with interest added, compounded monthly

Present Value – the amount that can be borrowed over a certain period, given the monthly payments and the compound interest rate.

Activity: Drag the black, dotted, vertical line and note the changing value of the **Balance Owing**.

Complete: Write down formulae for the following:

Compound Amount =

Future Value =

Present Value =

Use the applet to answer the following question:

What is the balance owing on a loan of R700 000 after 10 years with monthly payments of R7000, and an interest rate of 10% p.a. compounded monthly. _________________

Use the relevant formulae and your calculator to check this figure:

Use the applet to answer the following:

How long would it take to pay off a loan of R700 000 with monthly payments of R7000, and an interest rate of 15% p.a. compounded monthly? ________________

Vocabulary:

instalment:____

bond:____